UNIVERSITY OF CAMBRIDGE

Physics-enhanced velocimetry (PEV) for joint reconstruction and segmentation of noisy velocity images

Matthew Juniper, Alexandros Kontogiannis

Engineering Department, University of Cambridge, United Kingdom

Trustworthy AI, Maxwell Centre, Cambridge

Physics-enhanced velocimetry (PEV)

Alexandros Kontogiannis
(Ph.D. candidate)

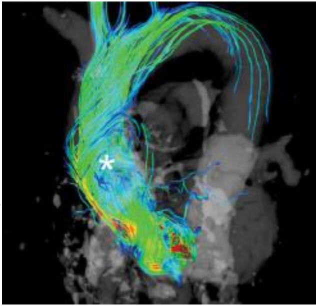
Prof. Matthew Juniper
(Supervisor)

The material of this presentation is taken from

A. Kontogiannis, S. V. Elgersma, A. J. Sederman, M. P. Juniper, ***Joint reconstruction and segmentation of noisy velocity images as an inverse Navier–Stokes problem***, JFM, 944 (A40) 2022.

Inverse problems in haemodynamics and PC-MRI

- PC-MRI measures 4D velocity fields $\boldsymbol{u}^{\star}$.
- Measurements become increasingly noisy as spatial resolution is increased.
- To increase SNR, repeated scans are averaged, leading to long signal acquisition times.
- PC-MRI signals often need reconstruction.

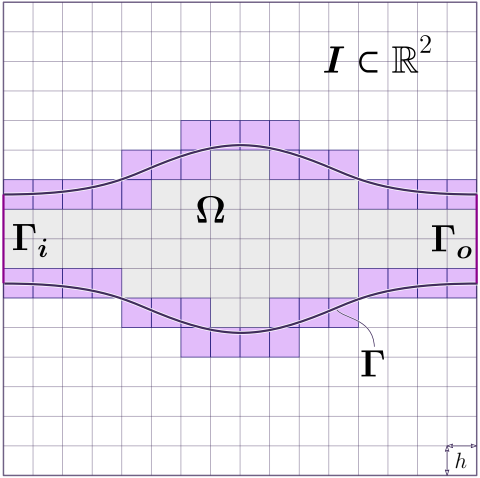


[Allen et al., J Cardiovasc Magn Reson 2014]

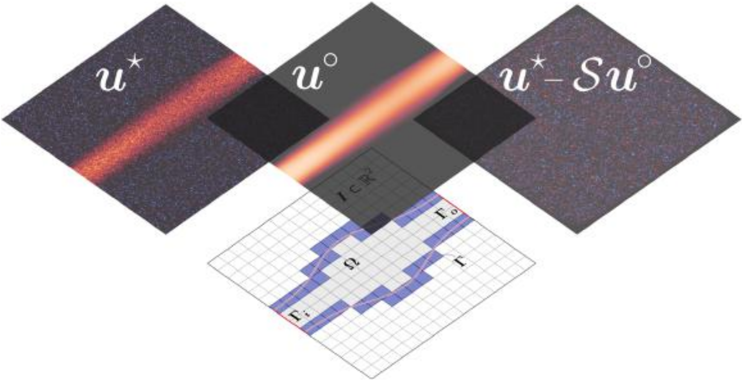
What do we do?

- We use prior information that the image is of a flow through a tube.
- We combine adjoint methods and shape optimization within a Bayesian framework to solve an inverse Navier–Stokes boundary value problem.
- We infer the *most likely* boundary position, velocity field and viscosity from a PC-MRI signal, obtaining the pressure and wall shear stress for free.

The Navier–Stokes boundary value problem (e.g. in 2D)



The inverse Navier–Stokes boundary value problem

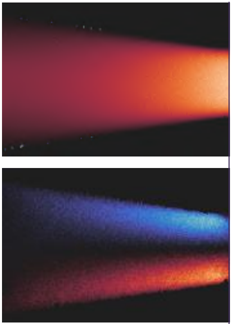


$\boldsymbol{u}^\star$	: noisy velocity image
$\boldsymbol{u}^\circ$	: reconstruction using an inv. N–S problem (digital twin)
$\boldsymbol{u}^\star - \mathcal{S}\boldsymbol{u}^\circ$	: noise/artefacts that we filter out
$\boldsymbol{\Gamma}$	: most likely boundary of the object Ω

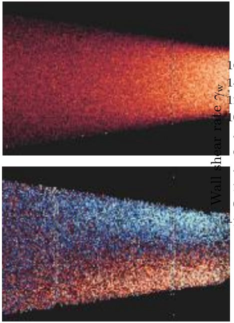
Denoising and improved wall-shear rate estimation

- We address two shortcomings of PC-MRI:
  i. SNR decreases as spatial resolution increases,
  ii. partial volume effects reduce signal near the boundaries,

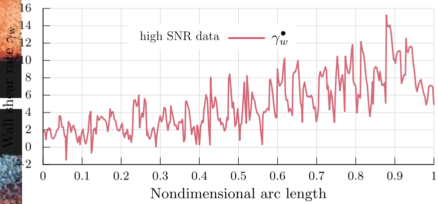
  which hinder the accurate estimation of **wall shear stresses**.



High SNR
PC-MRI images



Low SNR
PC-MRI images
×28 times faster

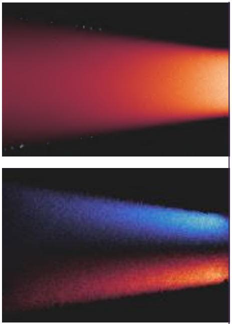


Even with high SNR data
wall shear rate is badly
approximated

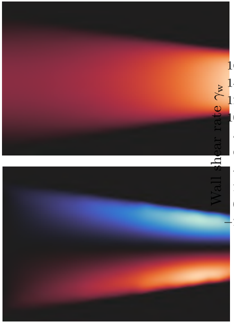
Denoising and improved wall-shear rate estimation

- We address two shortcomings of PC-MRI:
  i. SNR decreases as spatial resolution increases,
  ii. partial volume effects reduce signal near the boundaries,

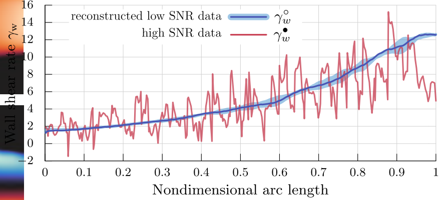
  which hinder the accurate estimation of **wall shear stresses**.



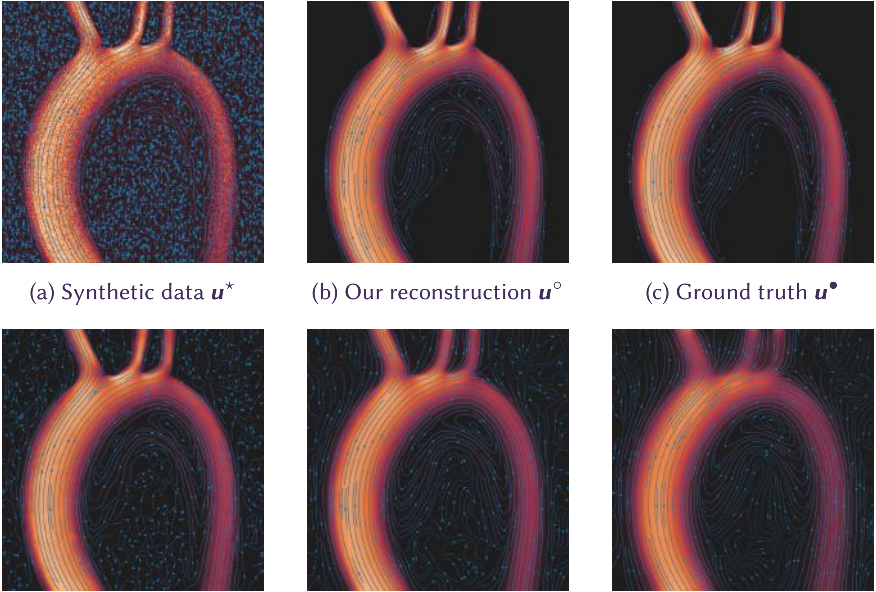
High SNR
PC-MRI images



Reconstructed low SNR
PC-MRI images
$\times$28 times faster



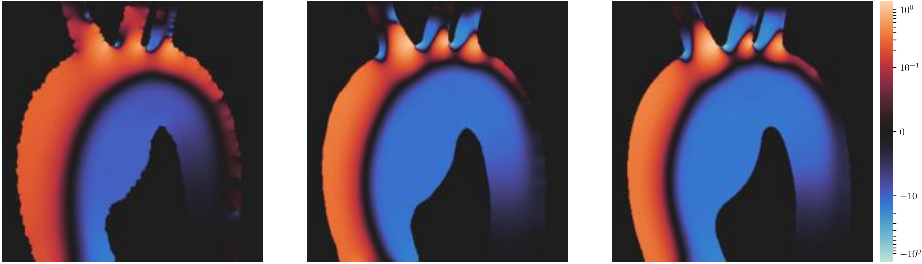
Reconstructed low SNR data
approximate better the wall
shear stress



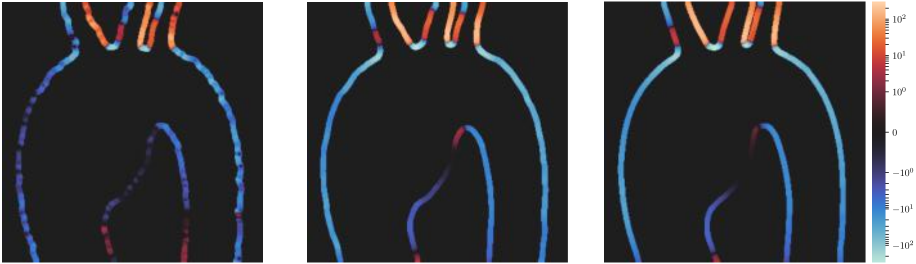
(a) Synthetic data $\boldsymbol{u}^\star$ (b) Our reconstruction $\boldsymbol{u}^\circ$ (c) Ground truth $\boldsymbol{u}^\bullet$

(d) TV-B $\lambda/\lambda_0 = 0.1$ (e) TV-B $\lambda/\lambda_0 = 0.01$ (f) TV-B $\lambda/\lambda_0 = 0.001$

Figure: Streamlines in the simulated 2D model of an aortic aneurysm ($\mathrm{Re} = 500$).



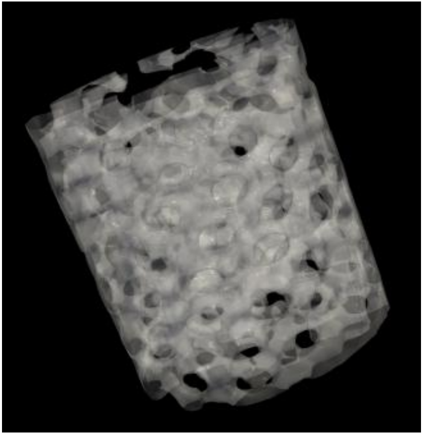
(a) Zeroth iteration p_0 (b) Our reconstruction p° (c) Ground truth $p^\bullet$



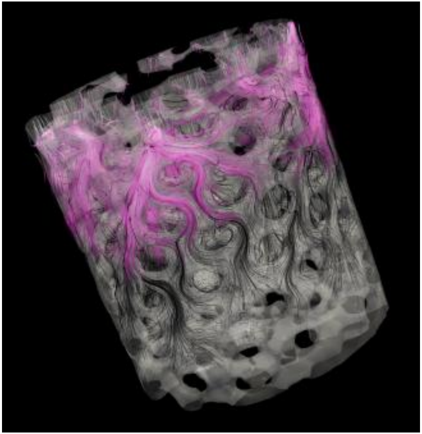
(d) Zeroth iteration $(\gamma_w)_0$ (e) Our reconstruction γ_w° (f) Ground truth $\gamma_w^\bullet$

Figure: Inferred wall shear stress and pressure.

Work in progress (porous media flows)



(a) Segmented geometry of a packed bed

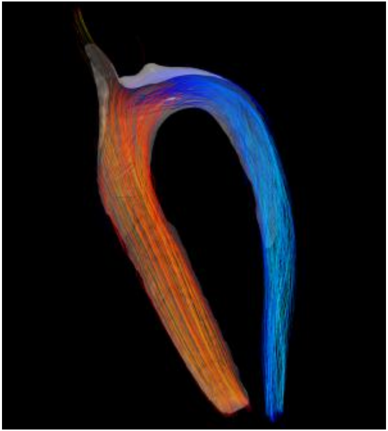


(b) Simulated flow through the packed bed

Figure: Demonstration of the parallel 3D Navier–Stokes (immersed-boundary finite element) solver that we have developed in a packed bed geometry.

Work in progress (cardiovascular flows)

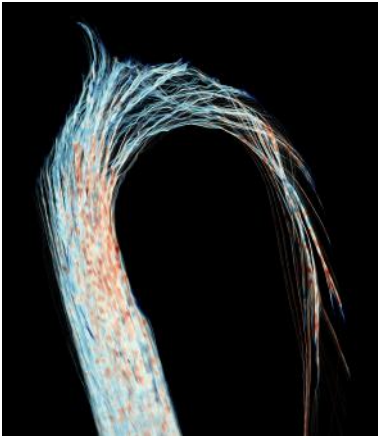
(a) Segm. geom. of an aortic arch replica



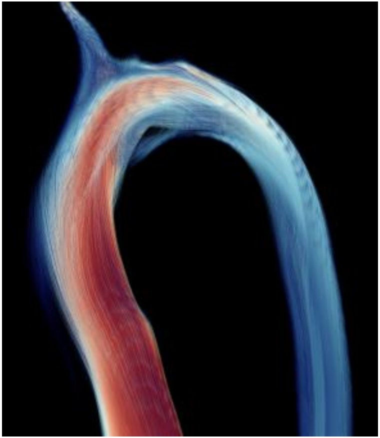
(b) PC-MRI of flow through the replica

Figure: High-quality PC-MRI experiment for the flow through an aortic arch replica (3D-printed model of an aortic arch scanned with CT).

Work in progress (cardiovascular flows)



(a) 3D experimental (noisy/sparse) PC-MRI



(b) 3D simulated aorta flow

Figure: 3D steady flows in complex geometries from (a) PC-MRI experiments and (b) the Navier–Stokes boundary value problem.

Conclusions

Summary

- We solve an inverse Navier–Stokes boundary value problem for the reconstruction of PC-MRI signals and simultaneously infer the wall-shear stress and pressure.

Why is this trustworthy?

- The data is assimilated into a model that is hard-wired to satisfy the Navier–Stokes equations. Uncertainties are rigorously quantified. There is no image processing, no neural network and no training data.
- The Navier–Stokes model can be complemented by viscosity models for blood, although this is unlikely to be necessary for vessels visible with MRI.
- Compared with current methods, this method requires around 100 times less data to extract the same flow information.

What's next?

- We will extend the method to 3D geometries and 4D periodic flows.
- We will reconstruct PC-MRI data of *in-vivo* cardiovascular flows.